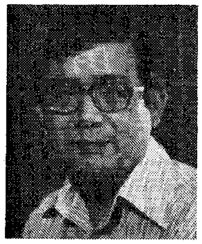

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# A Method for Diminishing Total Transmission Losses in Curved Dielectric Optical Waveguides

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***Abstract*—A practical method for diminishing total transmission losses in curved dielectric optical slab waveguides is proposed. Asymmetric structures are introduced into curved sections. It is found that there exists an optimum asymmetric structure for the curved section which makes the total transmission loss minimum. And it is also found that the characteristics of total transmission loss do not critically depend upon the asymmetry of waveguide structure, so that some displacement from the optimum structure does not increase the loss in an appreciable amount.**

## I. Introduction

PROPAGATING modes along a circular bend of dielectric waveguide suffer from a pure bending loss which mainly depends upon waveguide structure and curvature radius. In addition, a transition loss due to transformed radiation modes is incurred as soon as a propagating mode passes through a junction where two waveguides of different curvature radii are connected. Up to date, these two types of losses in several kinds of dielectric waveguides have been analyzed in detail separately [1]–[5]. A few reports have treated a total transmission loss which must be evaluated by taking both kinds of losses into consideration [6], [7].

Generally speaking, these losses must be decreased to the smallest possible amount in the application of dielectric waveguides to optical integrated circuits. Therefore, the loss reduction is an important problem which must be solved quickly from the practical point of view. To the authors' knowledge, however, it seems that methods for diminishing the total transmission loss have been scarcely presented.

Taking account of the well-known fact that the characteristics of each kind of loss are strongly affected by the waveguide structure, we can expect the improvement in the total transmission-loss characteristics provided that the waveguide structure is controlled appropriately. In the present paper, a practical method is proposed for diminishing the total transmission loss of a step-index dielectric optical slab waveguide containing a circular bend. An asymmetric index distribution is introduced into a curved section which connects two straight waveguides of symmetric index distribution. Though a very simple and easy method, it has an excellent effect on the total transmission loss as shown in the following sections.

The analysis is based on the assumption that the total transmission loss is given by the sum of pure bending loss and transition loss. This assumption may be good when the curved section is not perturbed and long enough compared with the wavelength of light. The pure bending loss is calculated by using the convenient approximate method presented by Marcuse [2]. The transition loss at the junction is estimated by the overlap integral of the wave functions in both sections. The perturbation solution is used for the wave function of curved section [8].

## II. Pure Bending Loss

A curved dielectric optical slab waveguide and a cylindrical coordinate system are shown in Fig. 1. The center of curvature coincides with the $z$ axis of the coordinate sys-

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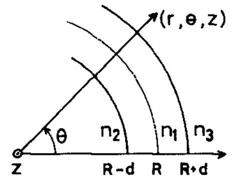


Fig. 1. Curved dielectric slab waveguide and cylindrical coordinate system.

tem. It is well known that the Bessel function with complex order is the solution of Maxwell's equations in the cylindrical coordinate system. The boundary condition that the tangential field components must be continuous at the dielectric interfaces leads to the eigenvalue equation whose solution is the complex propagation constant of the propagating mode. The pure bending loss is obtained from the imaginary part of it. The eigenvalue equation can be solved numerically without much trouble by using an electronic computer, if necessary.

On the other hand, several accurate approximate methods have been presented up to date [1]–[4], and they are much easier than the above rigorous procedure in the analysis of pure bending loss. In this paper, Marcuse's approximation will be used. The pure bending loss per unit length $2\alpha$ is expressed in the literature [2] as follows:

$$2\alpha=\frac{2\delta\kappa^2\exp\left[2\delta d-U\right]}{\left(n_1^2-n_3^2\right)k^2\beta\left(2d+\dfrac{1}{\gamma}+\dfrac{1}{\delta}\right)} \tag{1}$$

where

$$U=\left\{\frac{\beta}{\delta}\ln\left(\frac{1+\dfrac{\delta}{\beta}}{1-\dfrac{\delta}{\beta}}\right)-2\right\}\delta R \tag{2}$$

$$\kappa^2=n_1^2k^2-\beta^2 \tag{3}$$

$$\gamma^2=\beta^2-n_2^2k^2 \tag{4}$$

$$\delta^2=\beta^2-n_3^2k^2. \tag{5}$$

The propagation constant $\beta$ must be the solution of an eigenvalue equation

$$\tan 2\kappa d=\frac{\kappa(\gamma+\delta)}{\kappa^2-\gamma\delta}. \tag{6}$$

In the preceding equations, $2d$ is the slab thickness and $R$ is the curvature radius of waveguide center; $n_1$, $n_2$, and $n_3$ are the refractive indexes in the region $r<R-d$, $R-d<r<R+d$, and $R+d<r$, respectively; and $k$ is the wavenumber in free space. These equations are valid for TE modes, so that the following discussions will be restricted to those of the TE mode.

Let us study the dependence of pure bending loss upon the asymmetry in the waveguide structure. The numerical results for the lowest order mode with the aid of (1) are shown in Figs. 2 and 3, where the contribution of asymmetry in the refractive-index distribution is illustrated. In both figures, the ordinate is the normalized pure bending

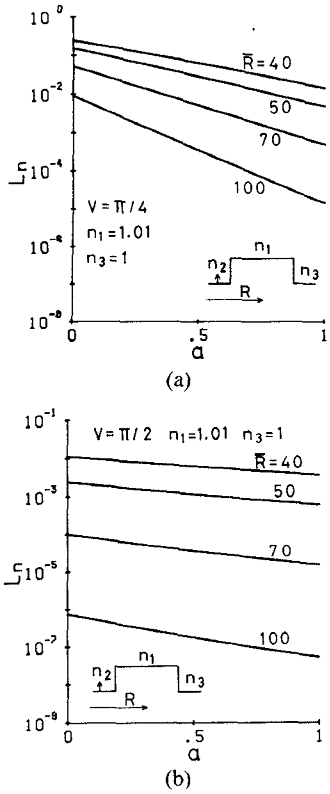


Fig. 2. Pure bending loss of the $TE_0$ mode versus asymmetry index of the waveguide structure with normalized curvature radius as a parameter, where $n_1=1.01$ and $n_2\geqslant 1=n_3$. (a) $V=\pi/4$. (b) $V=\pi/2$.

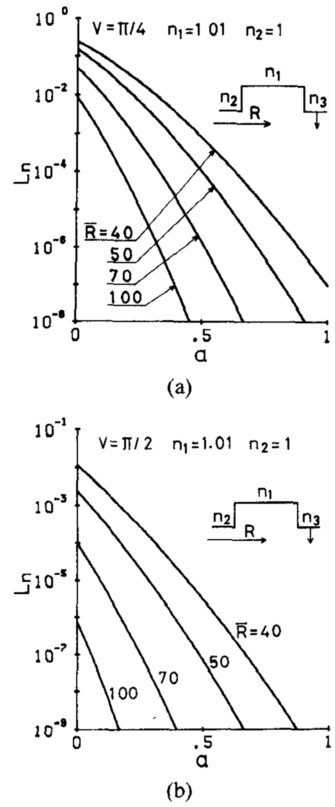


Fig. 3. Pure bending loss of the $TE_0$ mode versus asymmetry index of the waveguide structure with normalized curvature radius as a parameter, where $n_1=1.01$ and $n_2=1\geqslant n_3$. (a) $V=\pi/4$. (b) $V=\pi/2$.

loss per unit radian

$$L_n = \alpha R \left( \frac{n_1^2 - n_f^2}{n_1^2} \right)^{1/2} \tag{7}$$

and the abscissa is the asymmetry index

$$a = \frac{n_2^2 - n_3^2}{n_1^2 - n_2^2} \tag{8}$$

where

$$n_f = \text{fix}(n_2, n_3) \tag{9}$$

means that $n_f$ is equal to either $n_2$ or $n_3$ which is fixed in each case. The parameters $\bar{R}$ and $V$ are the normalized radius of curvature

$$\bar{R} = 2n_1 k \left( 1 - \frac{n_f^2}{n_1^2} \right)^{3/2} R \tag{10}$$

and the normalized frequency

$$V = kd\left(n_1^2 - n_f^2\right)^{1/2} \tag{11}$$

respectively. In Fig. 2, it is assumed that refractive indexes $n_1 = 1.01$, $n_f = n_3 = 1$, and $n_2$ increases from unity so that the asymmetry index changes from zero to unity, whereas in Fig. 3, $n_1 = 1.01$, $n_f = n_2 = 1$, and $n_3$ decreases from unity so that $a$ changes from zero to unity. Therefore, the waveguide is assumed to be weakly guiding. The other parameters have been chosen so that $\bar{R} = 40, 50, 70, 100$ in every figure, $V = \pi/4$ in (a), and $V = \pi/2$ in (b).

It is explicitly illustrated in Figs. 2 and 3 that the pure bending loss decreases monotonically with the increase of asymmetry index. The change of $n_3$ contributes much more efficiently than that of $n_2$. This is the property inherent in the pure bending loss.

## III. Transition Loss

When two waveguides of different curvature radius and/or structure are connected to each other, the propagating mode supported by one waveguide transforms to every possible mode of the other waveguide at the junction. The radiation loss due to the mode conversion is looked on as the transition loss. In the present case of the single-mode operation, the transition loss can be estimated in terms of the power transmitted through the junction, because the reflected power from it must be negligibly small under the weakly guiding condition. The transmission coefficient of the power through the junction can be easily obtained from the following overlap integral:

$$T = \frac{\left\{ \int E_1(u) E_2(u)\, du \right\}^2}{\int E_1(u)^2\, du \cdot \int E_2(u)^2\, du} \tag{12}$$

where $E_1(u)$ and $E_2(u)$ are the wave functions for the lowest order mode of two connected waveguides.

With the aid of (12), let us estimate the transmission coefficient in the case shown in Fig. 4 where a symmetric straight slab waveguide is connected to an asymmetric curved one. A perturbation solution

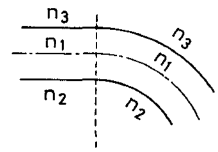


Fig. 4. Junction between symmetric straight slab waveguide and asymmetric curved slab waveguide, where $n_1 > n_2 = n_3$ in the straight section, and $n_1 > n_2 \geq n_3$ in the curved section.

$$E(u) = E_0(u) + \frac{F(u)}{R} \tag{13}$$

where

$$u = r - R \tag{14}$$

$$E_0(u) = \begin{cases} \cos(\kappa d + \psi) \exp[-\delta(u-d)], & u > d \\ \cos(\kappa u + \psi), & |u| < d \\ \cos(\kappa d - \psi) \exp[\gamma(u+d)], & u < -d \end{cases} \tag{15}$$

$$F(u) = \begin{cases} \dfrac{\beta^2 d}{2\kappa^2} \cos(\kappa d + \psi) \left\{ \left(1 + \dfrac{1}{\gamma d}\right)\left(1 - \dfrac{(\kappa d)^2}{\delta d}\right) + \dfrac{(\kappa d)^2}{\gamma d \delta d} \dfrac{u}{d} + \dfrac{(\kappa d)^2}{\delta d} \left(\dfrac{u}{d}\right)^2 \right\} \exp[-\delta(u-d)], & u > d \\ \dfrac{\beta^2 d}{2\kappa^2} \left\{ \kappa d \left[ \left(1 + \dfrac{1}{\gamma d}\right)\left(1 + \dfrac{1}{\delta d}\right) - \left(\dfrac{1}{\gamma d} - \dfrac{1}{\delta d}\right) \cdot \left(\dfrac{u}{d}\right) - \left(\dfrac{u}{d}\right)^2 \right] \cdot \sin(\kappa u + \psi) - \dfrac{u}{d} \cos(\kappa u + \psi) \right\}, & |u| < d \\ \dfrac{\beta^2 d}{2\kappa^2} \cos(\kappa d - \psi) \left\{ -\left(1 + \dfrac{1}{\delta d}\right)\left(1 - \dfrac{(\kappa d)^2}{\gamma d}\right) + \dfrac{(\kappa d)^2}{\gamma d \delta d} \dfrac{u}{d} - \dfrac{(\kappa d)^2}{\gamma d} \left(\dfrac{u}{d}\right)^2 \right\} \exp[\gamma(u+d)], & u < -d \end{cases} \tag{16}$$

$$\psi = \frac{1}{2} \tan^{-1} \frac{\kappa(\delta - \gamma)}{\kappa^2 + \gamma\delta} \tag{17}$$

is used for the wave function of curved slab waveguide [8]. Figs. 5 and 6 show the variation of transmission coefficient with the asymmetry index. In Fig. 5, it is assumed that $n_1 = 1.01$, $n_2 = n_3 = 1$ for the straight section, and $n_1 = 1.01$, $n_2 \geq 1$, $n_3 = 1$ for the curved section, whereas in Fig. 6, the straight section is the same as that in Fig. 5 and $n_1 = 1.01$, $n_2 = 1$, $n_3 \leq 1$ for the curved section. The other parameters have been chosen so that $\bar{R} = 40, 50, 70, 100$ in every figure, $V = \pi/4$ in (a), and $V = \pi/2$ in (b). The transmission coefficients obtained from (12) are in good agreement with those

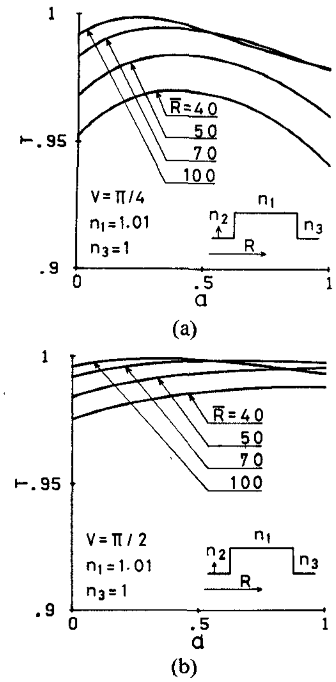


Fig. 5. Transmission coefficient of the $TE_0$ mode at the junction versus asymmetry index of the waveguide structure in curved section with normalized curvature radius as a parameter, where $n_1=1.01$ and $n_2=n_3=1$ in the straight section, and $n_1=1.01$ and $n_2 \geqslant 1 = n_3$ in the curved section. (a) $V=\pi/4$. (b) $V=\pi/2$.

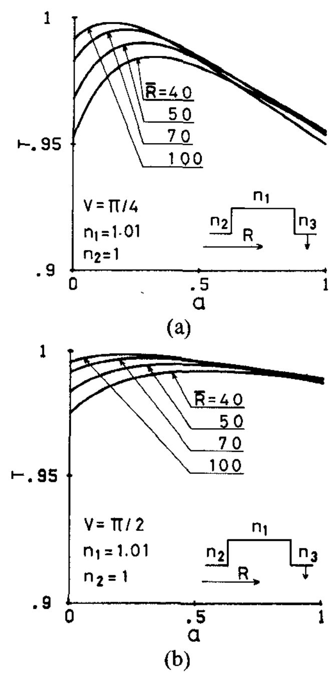


Fig. 6. Transmission coefficient of the $TE_0$ mode at the junction versus asymmetry index of the waveguide structure in curved section with normalized curvature radius as a parameter, where $n_1=1.01$ and $n_2=n_3=1$ in the straight section, and $n_1=1.01$ and $n_2=1 \geqslant n_3$ in the curved section. (a) $V=\pi/4$. (b) $V=\pi/2$.

of Taylor's analysis [5] for the case that $n_1=1.01$, $n_2=n_3=1$ in both straight and curved sections and $\bar{R}>30$.

From these figures, it is found that each curve certainly takes the maximum value at a certain asymmetry index for any curvature radius. The field distribution of curved section shifts from that of straight section to the far side of curvature center with decreasing curvature radius. On the other hand, the field of dielectric waveguide has a tendency to concentrate on the higher side of the refractive index when the waveguide structure is asymmetric. At the maximum point of the transmission coefficient, it is considered that the contribution of both of the above effects must just offset each other. The smaller the curvature radius becomes, the larger asymmetry is required.

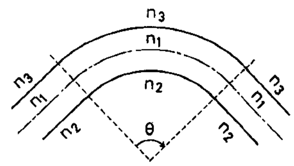


Fig. 7. Curved slab waveguide of turning angle $\theta$ connected to straight slab waveguides at both ends, where $n_1 > n_2 = n_3$ in the straight sections and $n_1 > n_2 \geqslant n_3$ in the curved section.

## IV. Total Transmission Loss

In the present section, the total transmission loss is estimated in the case that asymmetric curved slab waveguide of a certain curvature radius and arc length is connected to a symmetric straight slab waveguide at both ends as shown in Fig. 7. When each straight section consists of the same waveguide, the transmission coefficient at the entrance to the curved section is identical to that at the exit. Under the assumption that the total transmission loss can be obtained from the sum of pure bending loss and transition loss, the total transmission coefficient $T_{total}$ of the power passing through the whole curved section is expressed as

$$T_{total} = T \cdot \exp[-2\alpha R\theta] \cdot T \qquad (18)$$

where $\theta$ is the turning angle of curved section. In the following numerical examples, $\theta$ has been chosen as 90° in every case excepting Fig. 10.

Figs. 8 and 9 show the change of total transmission coefficient with the asymmetry index. In Fig. 8, it is assumed that $n_1=1.01$, $n_2=n_3=1$ for both straight sections and $n_1=1.01$, $n_2 \geqslant 1$, $n_3=1$ for the curved section, whereas in Fig. 9, the straight sections are the same as those in Fig. 8 and $n_1=1.01$, $n_2=1$, $n_3 \leqslant 1$ for the curved section. The other parameters have been chosen so that $\bar{R}=40$, 50, 70, 100 in every figure, $V=\pi/4$ in (a), and $V=\pi/2$ in (b).

It is illustrated in these figures that there exists the optimum asymmetry index which makes the total transmission coefficient maximum for any curvature radius as well as in Figs. 5 and 6. However, the asymmetry index of the maximum point is somewhat larger than that of the corresponding example for the transition loss. The displacement becomes larger with a decreasing curvature radius, because the contribution of pure bending loss becomes more dominant. It is one of the most practically important results obtained here that a great improvement in the total transmission coefficient can be achieved by introducing an

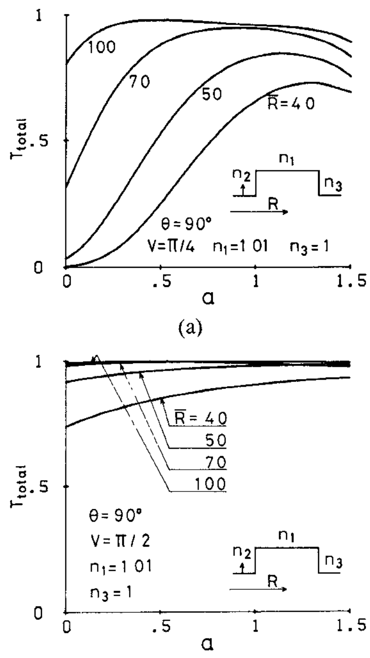


Fig. 8. Total transmission coefficient of the $TE_0$ mode versus asymmetry index of the waveguide structure in curved section with normalized curvature radius as a parameter, where $n_1=1.01$, and $n_2=n_3=1$ in the straight section and $n_1=1.01$, $n_2\geqslant 1=n_3$, and $\theta=90°$ in the curved section. (a) $V=\pi/4$. (b) $V=\pi/2$.

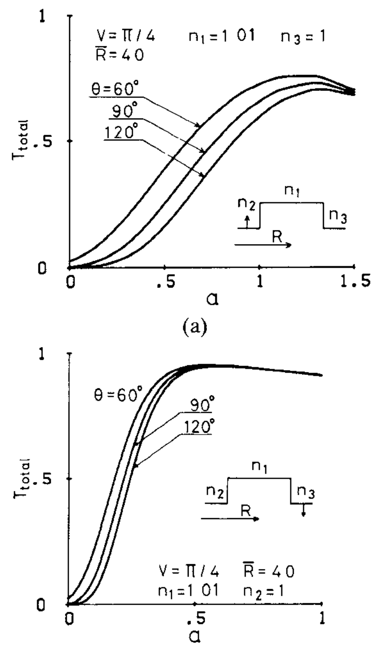


Fig. 10. Total transmission coefficients of the $TE_0$ mode versus asymmetry index of the waveguide structure in curved section with turning angle in curved section as a parameter, where $n_1=1.01$, and $n_2=n_3=1$ in the straight section, $\bar{R}=40$, and $V=\pi/4$. (a) $n_1=1.01$ and $n_2\geqslant 1=n_3$ in the curved section. (b) $n_1=1.01$ and $n_2=1\geqslant n_3$ in the curved section.

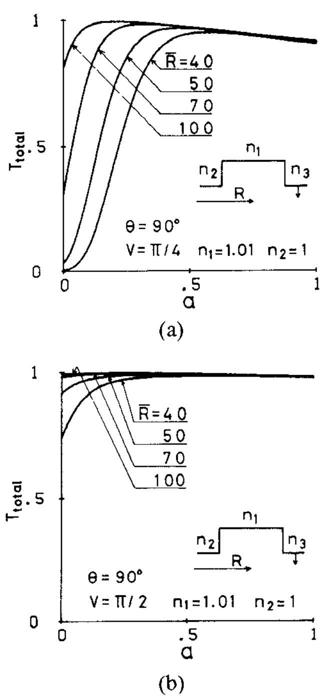


Fig. 9. Total transmission coefficient of the $TE_0$ mode versus asymmetry index of the waveguide structure in curved section with normalized curvature radius as a parameter, where $n_1=1.01$ and $n_2=n_3=1$ in the straight section and $n_1=1.01$, $n_2=1\geqslant n_3$, and $\theta=90°$ in the curved section. (a) $V=\pi/4$. (b) $V=\pi/2$.

adequate asymmetric structure into the curved section even though the asymmetry index is not just the optimum value.

In the comparison between the results of Figs. 8 and 9, it is apparent that the control of $n_3$ is more effective than that of $n_2$. For a typical example, in the case of $\bar{R}=40$ in Fig. 8(a), the transmission coefficient is about 0.73 at the maximum position. On the other hand, in Fig. 9(a) it is about 0.95 at the maximum position and is above 0.9 in such a wide range of asymmetry index as $0.4<a<1$. In the practical application, however, there may be the case in which it is very difficult to decrease the refractive index $n_3$ or something equivalent in the corresponding region. In such a case, a larger curvature radius is needed to ensure the required total transmission coefficient.

Finally, the change of the total transmission coefficient with the asymmetry index is illustrated in Fig. 10 with the turning angle of the curved section as a parameter. It is assumed that $\theta=60°$, $90°$, $120°$, $V=\pi/4$, $\bar{R}=40$ in both cases, $n_1=1.01$, $n_2\geqslant 1$, $n_3=1$ in (a), and $n_1=1.01$, $n_2=1$, $n_3\leqslant 1$ in (b). It is an important feature that the optimum asymmetry index scarcely changes for three arc lengths of the curved section in both cases shown in the figure.

Much more complicated structures exist in the actual waveguide with a three-dimensional guiding property for optical integrated circuits such as rib waveguides, optical strip waveguides, etc. However, asymmetric structures can be easily introduced into these waveguides by means of decreasing the film thickness of either side of the core in the case of rib waveguides and loading another material on either side of the loaded strip in the case of optical strip waveguides. These waveguides can be transformed into the equivalent slab waveguides by the concept of effective index in good approximation, so that the asymmetry of waveguide structure is included in the asymmetry of refractive index distribution in the equivalent slab waveguides.

Therefore, the results on slab waveguides obtained here can be directly applied to the actual waveguides for optical integrated circuits.

## V. Conclusions

The asymetric structure is introduced into the curved section to minimize the total transmission loss involving both the contribution of pure bending loss and transition loss in a dielectric slab waveguide. The analysis is based on the assumption that the total transmission loss is obtained from the sum of the pure bending loss and the transition loss.

As a result, it is shown that there exists an optimum asymmetric structure of waveguide for the curved section which makes the total transmission loss minimum when both the curvature radius and the arc length are fixed, and that the improvement of loss characteristics becomes pronounced more and more with a decreasing curvature radius. And it is also shown that the characteristics of the total transmission loss do not critically depend upon the asymmetry index of the waveguide structure, so that some deviation of the asymmetry index from its optimum value will not increase the loss in an appreciable amount. The results on slab waveguides obtained here would be still applicable to the actual waveguides for optical integrated circuits such as rib waveguides or optical strip waveguides without major modification.

## References

[1] E.A.J. Marcatili, "Bends in optical dielectric guides," *Bell Syst. Tech. J.*, vol. 48, pp. 2103–2132, Sept. 1969.

[2] D. Marcuse, "Bending loss of the asymmetric slab waveguide," *Bell Syst. Tech. J.*, vol. 50, pp. 2551–2563, Oct. 1971.

[3] S. Kawakami, M. Miyagi, and S. Nishida, "Bending losses of dielectric slab optical waveguide with double or multiple claddings: Theory," *Appl. Opt.*, vol. 14, pp. 2588–2597, Nov. 1975.

[4] L. Lewin, "Radiation from curved dielectric slabs and fibers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 718–727, July 1974.

[5] H. F. Taylor, "Power loss at directional change in dielectric waveguides," *Appl. Opt.*, vol. 13, pp. 642–647, Mar. 1974.

[6] M. Heiblum, "Analysis of curved optical waveguides by conformal transformation," *IEEE J. Quantum Electron.*, vol. QE-11, pp. 75–83, Feb. 1975.

[7] W. A. Gambling, H. Matsumura, C. M. Ragdale, and R. A. Sammut, "Measurement of radiation loss in curved single-mode fibres," *IEE J. Microwaves, Opt. Acoust.*, vol. 2, pp. 134–140, July 1978.

[8] Y. Takuma, M. Miyagi, and S. Kawakami, "New bending loss formula for dielectric slab waveguides," *Paper of Technical Group*, vol. OQE 80-78, pp. 69–74, Oct. 1980.

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